Multivariable Calculus
Quiz 4 **SOLUTIONS**

1) Compute $M\vec{v}$ for the given matrix M and vector $\vec{v}$.

a) $M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$

Solution: $M\vec{v} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$

b) $M = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

Solution: $M\vec{v} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}$

2) Consider the following matrices. For each of the operations below, either compute the result or state that the operation is not well-defined.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & -1 & 0 \end{bmatrix}$$

a) $A + B$

Solution: $A + B = \begin{bmatrix} 2 & 2 & 0 \\ 3 & 4 & 4 \end{bmatrix}$

b) BC

Solution: $BC = \begin{bmatrix} -2 & 2 & 1 \\ 13 & -3 & 4 \end{bmatrix}$

c) CB

Solution: This product is undefined (dimension mismatch between the matrices).

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3) Find the eigenvalues for the following matrix.

$$M = \begin{bmatrix} 2 & 1 \\ 9 & 2 \end{bmatrix}$$

Solution: The characteristic polynomial is

$$\begin{vmatrix} 2-\lambda & 1 \\ 9 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 9.$$

The roots of this polynomial are

$$\begin{aligned} (2-\lambda)^2 - 9 &= 0 \\ (\lambda - 2)^2 &= 9 \\ \lambda &= 2 \pm 3 = -1, 5. \end{aligned}$$

So, the eigenvalues are $\lambda = -1, 5$.

4) Given that $\lambda = 3$ is an eigenvalue for the following matrix, find the corresponding family of eigenvectors.

$$M = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$

Solution: We need to find solutions to $M\vec{v} = 3\vec{v}$ or $(M - 3\ \mathrm{Id})\vec{v} = \vec{0}$.

$$\left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -2 & 0 & 0 \\ -2 & 2 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This tells us that the components of any eigenvectors must satisfy $v_1 = -v_3$ and $v_2 = 0$. That means the corresponding family of eigenvectors for $\lambda = 3$ is given by

$$c \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$